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VELOCITIES

- USSR -

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MOTION OF THIN BODIES AT HIGH SUPERSONIC VELOCITIES¹⁾.

[Following is a translation of an article by G. M. Ben-Zelikovich, A. I. Bunimovich and M. P. Mikhaylove in the News of the Academy of Sciences USSR, Division of Technical Sciences, Moscow No. 1, 1960, pages 33-40.]
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Recently, along with the theoretical investigations of the ambient gas flow past bodies at high velocities, high-power experimentation facilities have been built which make it possible to study the problem of gas flow at speeds approaching the velocity of sound and considerably exceeding it.

The progress of the experiment has attracted the attention of the researchers to the problem of the establishment of similarity criteria in the ambient gas flow at high velocities past bodies which do not possess the property of geometrical similarity.

Ambient gas flow past thin bodies is investigated in Karmán's work [1] (velocities approaching the velocity of sound) and Tsien's [2] (high supersonic velocities), the flow being assumed to be plane or axially symmetrical, potential, and isentropic. The considerations of these authors are not rigorous because of the assumptions concerning potentiality and isentropy of the flow which clearly do not correspond to the physical properties of the flow.

In the present article, Tsien's results are generalized for the case of a three-dimensional motion in the presence of shock waves and vortexes, and it is shown that the problem on the steady

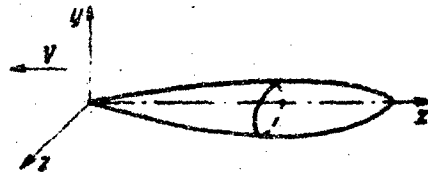


Figure 1.

1) The work was published for small circulation in the symposium "Theoretical Hydromechanics", No. 4, 1949. In a footnote to the work, it is pointed out that an authorized report of the papers [2,3] was presented in March 1948 at the seminar on hydromechanics at Moscow State University. The work is being reprinted without changes.

ambient gas flow past a thin body at a high supersonic velocity can be reduced approximately to the problem on unsteady gas motion in space with the number of dimensions being less by unity. Comparison of the results obtained and the available exact solutions determine the limits of the applicability of the approximation method.

1. Let us consider the motion of a thin body in the direction of the negative axis x at a constant velocity V which considerably exceeds the speed of sound (Figure 1).

Equations of the absolute motion of gas in projections on the fixed axes of coordinates x' , y' , z' have the following form.

Euler's equations

$$\begin{aligned}\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x'} + v \frac{\partial u}{\partial y'} + w \frac{\partial u}{\partial z'} &= -\frac{1}{\rho} \frac{\partial p}{\partial x'} \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x'} + v \frac{\partial v}{\partial y'} + w \frac{\partial v}{\partial z'} &= -\frac{1}{\rho} \frac{\partial p}{\partial y'} \\ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x'} + v \frac{\partial w}{\partial y'} + w \frac{\partial w}{\partial z'} &= -\frac{1}{\rho} \frac{\partial p}{\partial z'}\end{aligned}\quad (1.1)$$

Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x'} + \frac{\partial (\rho v)}{\partial y'} + \frac{\partial (\rho w)}{\partial z'} = 0 \quad (1.2)$$

Adiabatic equation

$$\frac{\partial}{\partial t} \frac{p}{\rho^\gamma} + u \frac{\partial}{\partial x'} \frac{p}{\rho^\gamma} + v \frac{\partial}{\partial y'} \frac{p}{\rho^\gamma} + w \frac{\partial}{\partial z'} \frac{p}{\rho^\gamma} = 0 \quad (1.3)$$

Here, p is pressure, ρ - density, u , v , w - velocity components of gas along axes x' , y' , z' .

Let us go over to the system of coordinates x , y , z connected rigidly with the moving body, after having made use of the transformation.

$$x' = x - Vt, \quad y' = y, \quad z' = z \quad (1.4)$$

Since the motion of gas relative to the body is steady, then for any function $F(x, y, z)$ the following relation is valid

$$\frac{\partial F(x, y, z)}{\partial x} V = \frac{\partial F(x' + Vt, y', z')}{\partial t} \quad (1.5)$$

Let b be the chord of the body and δ - some linear dimension characterizing the cross section (diameter of the cross section in the case of an axially symmetrical body; the median in the case of a plane

contour). Introducing dimensionless coordinates ξ, η, ζ according to formulas

$$x = b\xi, \quad y = \delta\eta, \quad z = \delta\zeta \quad (1.6)$$

and making use of the relation (1.5), we will obtain from the equations (1.1) - (1.3)

$$\begin{aligned} V \frac{\delta}{b} \frac{\partial u}{\partial \xi} + \frac{\delta}{b} u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \eta} + w \frac{\partial u}{\partial \zeta} &= -\frac{1}{\rho} \frac{\delta}{b} \frac{\partial p}{\partial \xi} \\ V \frac{\delta}{b} \frac{\partial v}{\partial \xi} + \frac{\delta}{b} u \frac{\partial v}{\partial \xi} + v \frac{\partial v}{\partial \eta} + w \frac{\partial v}{\partial \zeta} &= -\frac{1}{\rho} \frac{\partial p}{\partial \eta} \\ V \frac{\delta}{b} \frac{\partial w}{\partial \xi} + \frac{\delta}{b} u \frac{\partial w}{\partial \xi} + v \frac{\partial w}{\partial \eta} + w \frac{\partial w}{\partial \zeta} &= -\frac{1}{\rho} \frac{\partial p}{\partial \zeta} \\ V \frac{\delta}{b} \frac{\partial p}{\partial \xi} + \frac{\delta}{b} \frac{\partial (\rho u)}{\partial \xi} + \frac{\partial (\rho v)}{\partial \eta} + \frac{\partial (\rho w)}{\partial \zeta} &= 0 \\ V \frac{\delta}{b} \frac{\partial}{\partial \xi} \frac{p}{\rho^x} + \frac{\delta}{b} u \frac{\partial}{\partial \xi} \frac{p}{\rho^x} + v \frac{\partial}{\partial \eta} \frac{p}{\rho^x} + w \frac{\partial}{\partial \zeta} \frac{p}{\rho^x} &= 0 \end{aligned} \quad (1.7)$$

If the form of the body is defined by the equation

$$g(x/b, y/\delta, z/\delta) = g(\xi, \eta, \zeta) = 0 \quad (1.8)$$

then the ambient flow condition on the surface of the body may be written in the form

$$(V + u) \frac{\partial g}{\partial \xi} + v \frac{\partial g}{\partial \eta} + w \frac{\partial g}{\partial \zeta} = 0 \quad (1.9)$$

Fluid is at rest at the infinity ahead of the body

$$u = 0, \quad v = 0, \quad w = 0 \quad (1.10)$$

pressure equals p_0 and density equals ρ_0 .

When considering the motion of thin bodies at high velocities, we shall be neglecting terms of the form $(\delta/b) u (\partial u / \partial \xi), \dots, (\delta/b) \partial p / \partial \xi$ (which contain the small multiplier (δ/b) when comparison with the rest of the terms which are part of the equations (1.1) (1.2), (1.3). At the same time, terms containing multiplier $V \delta/b$, are retained since at high velocities V of the body, the magnitude of

$V(\delta/b)$ is finite.

Relating all velocities to the speed of sound in a fluid at rest, a_0 , for which

$$a_0^2 = \kappa \frac{p_0}{\rho_0} \quad (1.11)$$

(κ is the ratio of thermal capacities), denoting the dimensionless magnitudes of velocity, density, and pressure by

$$u^* = \frac{u}{a_0}, \quad v^* = \frac{v}{a_0}, \quad w^* = \frac{w}{a_0}, \quad p^* = \frac{p}{p_0}, \quad \rho^* = \frac{\rho}{\rho_0} \quad (1.12)$$

and introducing the dimensionless parameter

$$K = M_0 \frac{\delta}{b} \quad \left(M_0 = \frac{V}{a_0} \right) \quad (1.13)$$

we will obtain from equations (1.7)

$$\begin{aligned} K \frac{\partial u^*}{\partial \xi} + v^* \frac{\partial u^*}{\partial \eta} + w^* \frac{\partial u^*}{\partial \zeta} &= 0 \\ K \frac{\partial v^*}{\partial \xi} + v^* \frac{\partial v^*}{\partial \eta} + w^* \frac{\partial v^*}{\partial \zeta} &= -\frac{1}{\kappa \rho^*} \frac{\partial p^*}{\partial \eta} \\ K \frac{\partial w^*}{\partial \xi} + v^* \frac{\partial w^*}{\partial \eta} + w^* \frac{\partial w^*}{\partial \zeta} &= -\frac{1}{\kappa \rho^*} \frac{\partial p^*}{\partial \zeta} \\ K \frac{\partial p^*}{\partial \xi} + \frac{\partial (\rho^* v^*)}{\partial \eta} + \frac{\partial (\rho^* w^*)}{\partial \zeta} &= 0 \\ K \frac{\partial}{\partial \xi} + \frac{p^*}{\rho^*} + v^* \frac{\partial}{\partial \eta} \frac{p^*}{\rho^*} + w^* \frac{\partial}{\partial \zeta} \frac{p^*}{\rho^*} &= 0 \end{aligned} \quad (1.14)$$

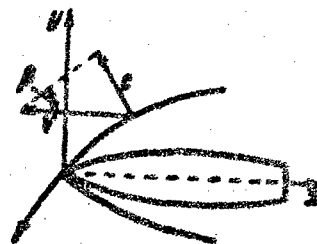


Figure 2

The ambient flow condition (1.9) will be rewritten in the following form after neglecting the magnitude of u upon comparison with V

$$K \frac{\partial g}{\partial \xi} + v^* \frac{\partial g}{\partial \eta} + w^* \frac{\partial g}{\partial \zeta} = 0 \quad (1.15)$$

Conditions at the infinity ahead of the body will take on the form

$$u^* = 0, \quad v^* = 0, \quad w^* = 0 \quad (1.16)$$

2. During the motion of a body at high supersonic velocity in a gas at rest, a shock wave originates which deflects little from the

surface of the body. If the normal component of the velocity of the shock wave propagation is denoted by c and the angle between the β direction of the tangent (Figure 2) to the shock wave and axis x is designated by α , then in view of the smallness of angle β we will obtain

$$c = V \sin \beta \approx V \beta \quad (2.1)$$

If v_t and v_n are the components of the velocity of gas along the tangent and the normal to the direction of the shock wave, then $v_t = u \cos \beta - v' \sin \beta$, or because β is small,

$$v_t \approx u$$

and v' may be regarded as the component of the velocity of gas in the direction perpendicular to axis x .

Conditions on the shock wave may be represented in the form

$$\begin{aligned} v_t &= 0, & p_1 &= p_0 \frac{2\alpha}{\alpha+1} \frac{c^2}{a_0^2} \left[1 - \frac{\alpha-1}{2\alpha} \frac{a_0^2}{c^2} \right] \\ v_n &= \frac{2}{\alpha+1} c \left[1 - \frac{a_0^2}{c^2} \right], & \rho_1 &= \rho_0 \frac{(\alpha+1)/(\alpha-1)}{1 + 2(a_0^2/c^2)/(\alpha-1)} \end{aligned} \quad (2.2)$$

where p_1 and ρ_1 are density and pressure beyond the shock wave.

Going to coordinates ξ, η, ζ , we will obtain from condition (2.1)

$$c = V \frac{\beta}{\alpha} \quad (2.3)$$

where α is magnitude determined in the process of the solution of the problem (the tangent of the angle between the direction of the tangent to the shock wave and axis ξ in the plane ξ, η).

Introducing parameter K into equations (2.2) and denoting

$$v_t^* = \frac{v_t}{a_0}, \quad v_n^* = \frac{v_n}{a_0}, \quad p_1^* = \frac{p_1}{p_0}, \quad \rho_1^* = \frac{\rho_1}{\rho_0} \quad (2.4)$$

we will obtain conditions on the shock wave in dimensionless form

$$\begin{aligned} v_t^* &= 0, & p_1^* &= \frac{2\alpha}{\alpha+1} K^2 \alpha^2 \left[1 - \frac{\alpha-1}{\alpha} \frac{1}{\alpha^2 K^2} \right] \\ v_n^* &= \frac{2}{\alpha+1} K \alpha \left[1 - \frac{1}{\alpha^2 K^2} \right], & \rho_1^* &= \frac{(\alpha+1)(\alpha-1)}{1 + \frac{2}{\alpha-1} \frac{1}{\alpha K}} \end{aligned} \quad (2.5)$$

It is clear from the equations and boundary conditions obtained that the only parameters of the problem are the magnitudes of K and x . Therefore, it is obvious that for pressure, density, and the components of the velocity of gas at any point of the flow, one may write

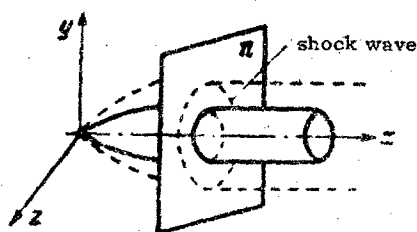
$$\begin{aligned} p^* = \frac{p}{p_0} &= f_1(K, x, \xi, \eta, \zeta), & \rho^* = \frac{\rho}{\rho_0} &= f_2(K, x, \xi, \eta, \zeta) \\ v^* = \frac{v}{a_0} &= f_3(K, x, \xi, \eta, \zeta), & w^* = \frac{w}{a_0} &= f_4(K, x, \xi, \eta, \zeta) \end{aligned} \quad (2.6)$$

3. It is easy to show that in such an approximate statement the problem on the steady motion of a thin body at a high supersonic velocity coincides with the problem on unsteady motion in space the number of dimensions of which is smaller by one.¹⁾

By the substitution of the variables in the equations (1.14)

$$t = \frac{b\xi}{V} = \frac{x}{M_0 a_0}, \quad y = \delta\eta, \quad z = \delta\zeta \quad (3.1)$$

we will obtain equations of the non-steady-state, two-dimensional problem in fixed plane π , perpendicular to the velocity of the body motion (Figure 3):



$$\begin{aligned} \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} \\ \frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} \\ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} &= 0 \\ \frac{\partial}{\partial t} \frac{p}{\rho^x} + v \frac{\partial}{\partial y} \frac{p}{\rho^x} + w \frac{\partial}{\partial z} \frac{p}{\rho^x} &= 0 \\ \frac{du}{dt} &= 0 \end{aligned} \quad (3.2)$$

Figure 3.

Condition (1.15) goes into the equation of the motion of the boundary which may be regarded as an unusual piston. Indeed, since

$$\frac{\partial g}{\partial \xi} = \frac{\partial g}{\partial t} \frac{b}{V}, \quad \frac{\partial g}{\partial \eta} = \frac{\partial g}{\partial y} \delta, \quad \frac{\partial g}{\partial \zeta} = \frac{\partial g}{\partial z} \delta \quad (3.2a)$$

1) The feasibility of reducing the problem on the three-dimensional steady motion (in this approximate statement) to the two-dimensional problem on unsteady motion is pointed by Hayes [3].

we will obtain from (1.15)

$$\frac{\partial g}{\partial t} + v \frac{\partial g}{\partial y} + w \frac{\partial g}{\partial z} = 0 \quad (3.3)$$

Conditions at infinity (1.16) go into initial conditions:

$$v = 0, \quad w = 0 \quad \text{upon } t = 0 \quad (3.4)$$

The change in δ under the condition that K remain constant is simply a change in the scale in the non-steady-state problem.

Thus, the problem on the motion of a thin body at high supersonic velocity in three-dimensional space corresponds in the approximate statement to the plane non-steady-state problem of the expansion of a cylindrical piston.

The plane problem on the motion of a thin body at a high supersonic velocity corresponds to one-dimensional problem on the unsteady motion of a piston in the presence of the shock wave.

4. In order to determine the limits of the applicability of the similarity criteria obtained, let us compare the approximate solution with the available exact solutions.

As the first example, let us consider the problem on the motion of a wedge having angle 2θ in the direction of axis x at supersonic velocity (Figure 4).

As is known, a straight-line shock wave forms in the flow, this shock wave originating at the point of the wedge and forming angle β with axis x .

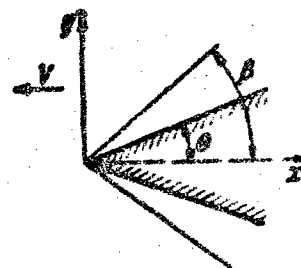


Figure 4

It can be derived from the conditions on the shock wave (2.2) that

$$\begin{aligned} \Theta &= \beta - \arctg \frac{[(x-1)M_0^2 \sin^2 \beta + 2] \tg \beta}{(x+1)M_0^2 \sin^2 \beta} \\ p_1 &= \frac{p_0}{x+1} [2xM_0^2 \sin^2 \beta - (x-1)] \end{aligned} \quad (4.1)$$

whence it is possible to find the relation p_1/p_0 as the function of the number M_0 and the central angle Θ of the cone. In the case being considered, parameter K equals

$$K = M_0^{\frac{2x}{x+1}} = 2M_0 \tg \Theta \quad (4.2)$$

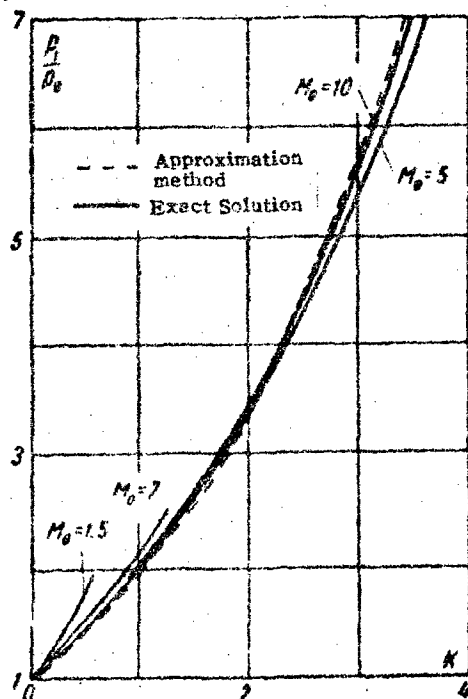


Figure 5

As is known, in piston motion the pressure and velocity on the piston equal the pressure and velocity beyond the shock wave.

Eliminating the magnitude a_0/c from the second and third equations of (2.2) and making use of (4.4), we will obtain

$$\frac{p_1}{p_0} = 1 + \frac{x(x+1)}{4} \left(\frac{K}{2}\right)^2 + x \frac{K}{2} \sqrt{1 + \frac{(x+1)^2}{16} \left(\frac{K}{2}\right)^2} \quad (4.5)$$

The curve corresponding to (4.5) is represented in Figure 5 by the broken line.

It is evident from Figure 5 that for large numbers for M_0 and small central angles of the wedge, the approximate solution coincides very well with the exact solution.

For comparison there are plotted in Figure 6 broken-line curves which correspond to the approximate computation of p_1/p_0 by Donovan's formula [4] if one confines oneself in it to the terms of the second order relative to θ . Donovan's formula has the form

$$\frac{p_1}{p_0} = 1 + \frac{xM_0^2}{2\sqrt{M_0^2-1}} K + x \frac{1-M_0^2 + 1/4(x+1)M_0^4}{4(M_0^2-1)^2} K^2 \quad (4.6)$$

Graphs of the relation of p_1/p_0 to $K(x=1.4)$ have been constructed (Figure 5) on the basis of the computation performed for numbers $M_0 = 1.5, 2.5, 10$ and variations of angle θ from 0 to 20°.

The approximate solution of the problem on wedge motion at high supersonic velocities amounts to the solution of one-dimensional problem on piston motion at a constant speed.

The speed of piston motion is obviously equal to the vertical velocity of the wedge, i.e.,

$$v = V \tan \theta \quad (4.3)$$

and consequently,

$$K = \frac{v}{a_0} 2 \tan \theta = 2 \frac{v}{a_0} \quad (4.4)$$

where $K = 2M_0\theta$ is taken approximately.

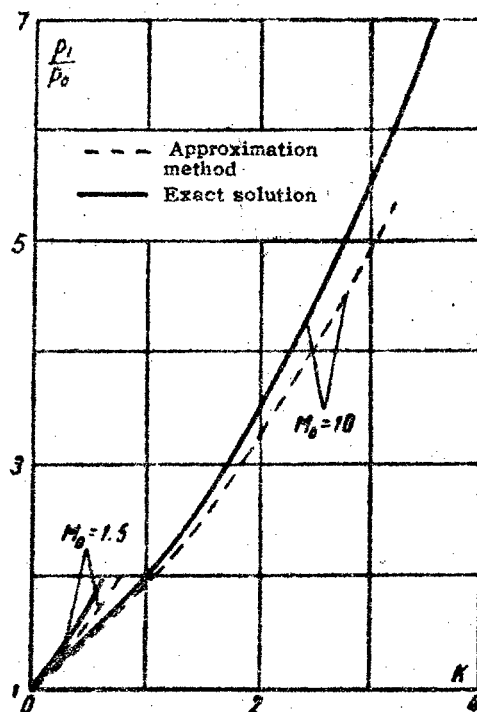


Figure 6.

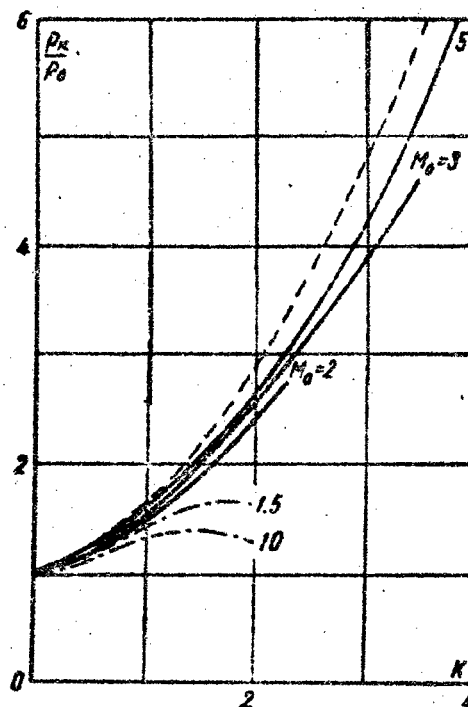


Figure 7.

As the second example, let us consider the motion of a right circular cone in the direction of its axis at a supersonic velocity.

The exact solution of this problem was given by Busemann [5] and the detailed computations were performed by Taylor and Mac Coll [6].

We showed above that this problem corresponds approximately to the problem on the expansion of a circular cylindrical piston at a steady velocity. And the solution of the problem on the expansion of a cylindrical piston was given L. I. Sedov [7].

In Figure 7 are given the curves of the relation between the ratio of the pressure p_K on the cone (cylindrical piston) to pressure p_0 in gas at rest, and the number for $K = (V/a_0)2\lg\theta = 2v/a_0$ (because, as in the case of the wedge, $\delta/b = 2\lg\theta$, and the velocity of piston motion is the vertical cone velocity equalling $V\lg\theta$). The broken line represents the result of numerical integration of equations derived in Sedov's work. Continuous lines show the dependence of p_K/p_0 on K with different numbers for V/a_0 , this relation having been computed on the basis of the exact solution by Taylor and Mac Coll. Dot-and-dash lines show the dependence of

p_K/p_0 on K with different numbers for V_0/a_0 , this relation having been obtained by Busemann's formula [8].

It is evident from Figure 7 that the difference in the values produced by the exact and approximate solutions decreases with the increase in the number for M_0 . For $M_0=3$, the result obtained from the approximate solution may be considered satisfactory when $K < 1$ (an error of less than 5%), i.e. for the central angles of the cone $2\theta < 18^\circ$. For $M_0=5$, the result is satisfactory up to $K=2$, i.e. for cones with central angles $2\theta < 22^\circ$. Generally, however, pressures computed by approximation method are greater in magnitude than pressures computed by the exact method. Conversely, pressures computed by linearized method are smaller in magnitude than pressures computed by the exact method, the error increasing with the increase in the number for M_0 .

5. As has been shown, with the assumptions made above concerning the form of the bodies and the flow velocity, all dimensionless dynamic elements of motion depend only on the dimensionless parameter K . Making use of this, let us find now the functional relationship for the coefficients of head-on drag c_x and lift force c_y .

Wing of Infinite Span. Let us define the equation of contour in the form

$$y = \delta H(\xi) = \delta H\left(\frac{x}{b}\right) \quad (5.1)$$

where $H(\cdot)$ is the distribution function of the wing thicknesses. Then, making use of (2.6), we will obtain for the aggregate drag and lift force

$$X = \int p \frac{dy}{dx} dx = \delta \int p H'(\xi) d\xi = \delta p_0 \Phi_1(K) \quad (5.2)$$

$$Y = \int p dx = b p_0 \int \frac{p}{p_0} d\xi = b p_0 \Phi_2(K) \quad (5.3)$$

Expressions for the coefficients of drag and lift force have the form

$$c_x = \frac{X}{(1/2)\rho_0 V^2 b} = 2 \frac{\delta}{b} \frac{p_0}{\rho_0} \frac{\Phi_1(K)}{V} = \frac{2}{K} \frac{a_0^2}{V^2} K \Phi_1(K) = \frac{F_1(K)}{M_0^2} \quad (5.4)$$

$$c_y = \frac{Y}{(1/2)\rho_0 V^2 b} = 2 \frac{p_0}{\rho_0} \frac{\Phi_2(K)}{V^2} = \frac{F_2(K)}{M_0^2} \quad (5.5)$$

For bodies similar in the sense indicated, Akkeret's linearized theory gives the coefficients of c_x and c_y in the form

$$c_x \sim \frac{(\delta/b)^2}{\sqrt{M_0^2 - 1}}, \quad c_y \sim \frac{(\delta/b)}{\sqrt{M_0^2 - 1}} \quad (5.6)$$

For large values of the number for M_0 , these expressions assume the form

$$c_x \approx \frac{(\delta/b)^2}{M_0} = \frac{K^2}{M_0^3}, \quad c_y \approx \frac{(\delta/b)}{M_0} = \frac{K}{M_0^3} \quad (5.7)$$

which agrees with the equations (5.4) and (5.5) although these equations are considerably more general.

Axially Symmetrical Body. Let the equation of the body be defined in the form

$$r = \delta h(\xi) = \delta h\left(\frac{x}{b}\right) \quad (5.8)$$

where δ is the maximum diameter of the cross section; $h(\xi)$ — the distribution function of thicknesses. Then, we will obtain for the total drag

$$X = \int p 2\pi r dr = 2\pi \delta^2 p_0 \int \frac{p}{p_0} h(\xi) h'(\xi) d\xi = 2\pi \delta^2 p_0 \Phi_3(K) \quad (5.9)$$

The coefficient of drag related to the area of the maximum cross section is

$$c_x = \frac{2\pi \delta^2 p_0 \Phi_3(K)}{(1/2)\rho_0 V^2 \cdot (1/4)\pi \delta^2} = \frac{F_3(K)}{M_0^3} \quad (5.10)$$

The similarity laws obtained, show that for bodies with the same distribution of thicknesses at the angles of attack proportional to the relation δ/b , amounts $c_x M_0^3$ and $c_y M_0^2$ (wing of infinite span), $c_x M_0^2$ (axially symmetrical body) will be functions of one parameter $K = M_0 \delta/b$.

Consequently, having experimental results of blowing or of

flight tests for some profile with different numbers for M_0 , it is possible to recompute the obtained results for a series of profiles with the same distribution of thicknesses. Conversely, after blowing upon a series of profiles, similar in the sense indicated, with the same number for M_0 , it is possible to recompute the results for each of the profiles of the series with different numbers for M_0 .

Formulas derived in the present article for the magnitudes of c_x and c_y after taking into account the presence of shock waves and vortexes in the flow, coincide with Tsien's results obtained with the assumption of potentiality and isentropy of the flow.

Submitted 26 August 1959

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